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2009 J. Phys. A: Math. Theor. 42 315303

(http://iopscience.iop.org/1751-8121/42/31/315303)

IOP PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND THEORETICAL

J. Phys. A: Math. Theor. **42** (2009) 315303 (17pp) doi:10.1088/1751-8113/42/31/315303

A Bose–Einstein condensate in a double-well and periodic potential: a variational study

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Received 25 February 2009, in final form 10 June 2009
Published 16 July 2009
Online at stacks.iop.org/JPhysA/42/315303

Abstract

The dynamics of a Bose–Einstein condensate under the influence of a confining double-well potential and a periodic potential (optical lattice) is studied by the well-known variational method. The variational principle leads to coupled nonlinear differential equations for the amplitude (width), chirp, center position and the center frequency of the condensate. The interplay between the condensate parameters leads to various regimes of its time evolution which depend on the relative strengths of the double-well and the periodic potentials. In particular, we show that by adjusting the strengths of the applied potentials appropriately, one can control and manipulate the condensate in several ways including tunneling from one well to the other and the to-and-fro motion between the two wells.

PACS numbers: 82.40, 03.75.Fi, 03.65.Ge, 03.75.Kk

(Some figures in this article are in colour only in the electronic version)

Introduction

After the experimental realization of Bose–Einstein condensates (BEC), macroscopic manifestation of a large number of quantum phenomena could be studied, using various confining potential configurations, which would not have been possible otherwise. Along with other interesting physical phenomena, the possibility of tunneling of atoms between the traps containing two Bose–Einstein condensates was predicted [1]. At present, the means by which one can control and manipulate a condensate is a central issue in the study of BEC matter waves. In this context, the system of coupled BECs in a double-well trap [1–9] is very promising. The phenomenon of coherent quantum tunneling between two Bose–Einstein condensates, confined in a double-well magnetic trap, was theoretically studied by several authors and its similarity with the well-known Josephson effect in superconductors was

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established [2–4]. The possibility of the experimental observation of bosonic Josephson-type oscillations and the nonlinear self-trapping of the condensate, in such a system, were also discussed [5, 7] and predicted. The phenomenon was subsequently observed experimentally [8, 9].

The Josephson-type behavior of a Bose–Einstein condensate has also been studied in a quasi-one-dimensional optical lattice [10–16] which acts as a source of an additional force on the condensate which can be controlled externally. These studies showed that, in addition to the Josephson-type oscillations, such a system exhibits observable resonances and multimode behavior which could be used in tailoring matter waves and trapping and manipulating BECs. Hence, an optical lattice, superposed on a trapping potential, not only allows one to study various interesting phenomena, such as interference and diffraction of matter waves [17, 18], quantum logic [19], formation of Mott structures in optical lattices [20, 21] etc, it also provides a tool for controlling and manipulating BECs [22, 23].

Recently, Berry and Kutz [24] have studied the dynamical behavior of a BEC under the influence of both the harmonic and periodic external potentials. Besides giving an insight into the dynamics originating from the interaction of various physical characteristics of the condensate (such as the amplitude, chirp, center position and the center frequency), their work showed the possibility of controlling and manipulating BECs with the help of an optical lattice superimposed on a harmonic trapping potential.

In the given work, following Berry and Kutz, we study the dynamics of a BEC in a double-well potential, supplemented by an optical lattice, variationally. We show that the interplay between the physical characteristics of the condensate leads to various regimes of time evolution that depend on the relative strengths of the double-well and the periodic potentials. In particular, we show that by adjusting the strengths of the applied potentials appropriately, one can control and manipulate the condensate in several ways including tunneling from one well to the other and the to-and-fro motion between the two wells. We also show that the condensate can be controlled, so that it remains in a desired well of the double-well potential.

The model

We consider a one-dimensional dilute BEC in a double well confining potential and an optical lattice. Such a system is described by the following dimensionless Gross–Pitaevski (GP) equation:

$$\mathrm{i}\psi_t(x,t) + \tfrac{1}{2}\psi_{xx}(x,t) + \alpha|\psi(x,t)|^2\psi(x,t) - V(x)\psi(x,t) = 0, \tag{1}$$

where $\psi(x,t)$ is the complex-valued macroscopic wavefunction of the condensate in the mean-field approximation, t is the dimensionless time, x is the dimensionless distance, and the subscripts t and x stand for the corresponding partial derivatives with respect t and x, respectively. Here, α is a dimensionless coefficient (see equation (3)). Note that $\alpha = +1$ corresponds to an attractive, while $\alpha = -1$ corresponds to a repulsive condensate. The potential $V(x)$

$$V(x) = V_1(x^2 - \lambda^2)^2 + V_0\sin^2[\omega(x-\bar{x})] \tag{2}$$

consists of two parts: the first term, $V_1(x^2-\lambda^2)^2$, represents the confining double-well potential while the second part, $V_0\sin^2[\omega(x-\bar{x})]$, gives the optical lattice. The non-dimensionalization has been done as follows. The condensate wavefunction ψ is normalized to $\sqrt{N_0}$ (N_0 being the total number of atoms in the condensate), the distance x is measured in the units of $\sqrt{\hbar/m\omega_\perp}$, where m is the mass of the condensate atom and $\omega_\perp$ is the transverse confining frequency. Further, we have

$$\alpha = -g/(\hbar N_0 \omega_\perp), \tag{3}$$

where g is the self-interaction coefficient of the condensate, and ω is the frequency of the periodic potential. λ is a dimensionless constant that determines the minima of the double-well potential. The parameters $\pm\,\bar{x}$ determine the offset of a minimum of the periodic potential with respect to the corresponding minimum of the double-well potential. Note that when

$$\bar{x} = \frac{n\pi}{\omega}, \tag{4}$$

where n is an integer, a minimum of the periodic potential coincides with the corresponding minimum of the double-well potential. The quantities V_0 and V_1 (measured in the units of $\hbar\omega_\perp$) give the relative strengths of the periodic and the double-well potentials, respectively. Note that if we take the experimental values of the frequencies involved in the magnetic trap of the MIT experiment [2] ($\nu_x = 745$ Hz, $\nu_y = 235$ Hz, and $\nu_z = 410$ Hz), then one unit of the dimensionless distance x corresponds to approximately 0.964 μm in real units.

This paper is organized as follows. We start with the study of a single condensate in a double-well potential and show how one can manipulate the condensate between the two wells by changing the initial conditions as well as other parameters of the condensate. Here, we also determine the condition under which the condensate can tunnel through the barrier from one well to the other. Next, we perturb the double-well potential by an optical lattice (sinusoidal potential) and study the condensate dynamics in the perturbed potential. We show how the optical lattice leads to a richer set of possibilities for manipulating the condensate at will by varying the relative strength of the double-well and the sinusoidal potentials.

The variational analysis

In the given section, we present the variational treatment of the problem which is based on the work of Berry and Kutz [24]. The Gross–Pitaevskii equation can be derived from the variational principle

$$\delta \int_{t_1}^{t_2} \mathrm{d}t\, L = \delta \int_{t_1}^{t_2} \mathrm{d}t \int_{-\infty}^{+\infty} \mathrm{d}x\, \mathcal{L}((\psi, \psi^*, \mathrm{d}\psi/\mathrm{d}t, \mathrm{d}\psi^*/\mathrm{d}t) = 0, \tag{5}$$

with the Lagrangian density

$$\mathcal{L} = \mathrm{i}\left(\frac{\partial\psi^*}{\partial t}\psi - \psi^*\frac{\partial\psi}{\partial t}\right) + \frac{1}{2}\left|\frac{\partial\psi}{\partial x}\right|^2 - \alpha|\psi|^4 + 2V(x)|\psi|^2, \tag{6}$$

where the asterix stands for complex conjugation. The governing equation (1) results from $\delta\mathcal{L}/\delta\psi^* = 0$. For further analysis, we choose the following Gaussian ansatz

$$\psi = A\sqrt{\eta}\exp[-\eta^2(x - x_0)^2 + \mathrm{i}\beta(x - x_0)^2 + \mathrm{i}\xi(x - x_0) + \mathrm{i}\phi], \tag{7}$$

where η, β, ξ, ϕ and x_0 vary with time and measure the amplitude, chirp, center frequency, the absolute phase and the center position, respectively. The constant A determines the total number of atoms in the condensate.

Condensate in a double-well potential

Here, we consider the dynamics of a BEC in a double-well trapping potential, described by

$$V(x) = V_1(x^2 - \lambda^2)^2. \tag{8}$$

For an attractive condensate ($\alpha = +1$), using the above ansatz, we compute the reduced Lagrangian $\tilde{L}$:

$$\tilde{L} = \sqrt{\frac{\pi}{2}}A^2\left[\frac{1}{2\eta^2}\frac{\mathrm{d}\beta}{\mathrm{d}t} + 2\frac{\mathrm{d}\phi}{\mathrm{d}t} - 2\xi\frac{\mathrm{d}x_0}{\mathrm{d}t} + (\eta^4+\beta^2)\frac{1}{\eta^2} + \xi^2\right] + \sqrt{2\pi}A^2V_1\left\{\frac{3x_0^2}{2\eta^2} - \frac{\lambda^2}{2\eta^2} + \left(x_0^2-\lambda^2\right) + \frac{3}{16\eta^4}\right\} - \frac{A^4}{2}\eta\sqrt{\pi}. \tag{9}$$

Applying the Euler–Lagrange equation for each of the parameters η, β, ξ, ϕ and x_0, we arrive at the following system of ordinary differential equations for the condensate parameters:

$$\frac{\mathrm{d}\eta}{\mathrm{d}t} = -2\beta\eta, \tag{10}$$

$$\frac{\mathrm{d}x_0}{\mathrm{d}t} = \xi, \tag{11}$$

$$\frac{\mathrm{d}\beta}{\mathrm{d}t} = 2V_1\left(\lambda^2 - \frac{3}{4\eta^2} - 3x_0^2\right) + 2(\eta^4-\beta^2) - \frac{A^2\sqrt{2}}{2}\eta^3, \tag{12}$$

$$\frac{\mathrm{d}\xi}{\mathrm{d}t} = 4V_1x_0\left(\lambda^2 - x_0^2\right) - \frac{3V_1x_0}{\eta^2}. \tag{13}$$

The above system of nonlinear ordinary differential equations governs the interaction between various parameters of the condensate. Note that, due to the phase invariance of equation (1) [24], the absolute phase $\phi(t)$ does not appear in the above system and hence is irrelevant for the condensate dynamics.

The fixed points of the above system are determined by the set of parameters η_0, β_0, ξ_0 and $x_0 = X_0$. Here, $\beta_0 = \xi_0 = 0$, and η_0 and X_0 are obtained from the solutions of the following system of algebraic equations:

$$\frac{\mathrm{d}\beta}{\mathrm{d}t} = 2V_1\left(\lambda^2 - \frac{3}{4\eta^2} - 3x_0^2\right) + 2\eta^4 - \frac{A^2\sqrt{2}}{2}\eta^3 \tag{14}$$

$$4V_1x_0\left(\lambda^2 - x_0^2\right) - \frac{3V_1x_0}{\eta^2} = 0. \tag{15}$$

Linear stability analysis of the fixed points

As usual, we look for small deviations from the fixed point and put

$$\eta = \eta_0 + \tilde{\eta}, \qquad \beta = 0 + \tilde{\beta}, \qquad x_0 = X_0 + \tilde{x}, \qquad \xi = 0 + \tilde{\xi}, \tag{16}$$

where $\tilde{\eta}$, $\tilde{\beta}$, $\tilde{x}$ and $\tilde{\xi}$ are small deviations from the respective steady state values. Inserting these into the system of equations (10)–(13), we obtain the following set of differential equations governing the time evolution of the perturbations

$$\frac{\mathrm{d}\tilde{x}}{\mathrm{d}t} = \tilde{\xi}, \tag{17}$$

$$\frac{\mathrm{d}\tilde{\xi}}{\mathrm{d}t} = \tilde{x}\left(-12X_0^2V_1 + 4\lambda^2V_1 - \frac{3V_1}{\eta_0^2}\right) + \tilde{\eta}\frac{6X_0V_1}{\eta_0^3}, \tag{18}$$

$$\frac{\mathrm{d}\tilde{\eta}}{\mathrm{d}t} = -2\eta_0\tilde{\beta}, \tag{19}$$

$$\frac{d\tilde{\beta}}{dt} = \tilde{x}(-12X_0V_1) + \tilde{\eta}\left(8\eta_0^3 - \frac{3\sqrt{2}A^2\eta_0^2}{2} + \frac{3V_1}{\eta_0^3}\right). \tag{20}$$

The above set of equations can be written in the following matrix form:

$$\frac{dY}{dt} = MY, \tag{21}$$

where M is a 4×4 square matrix,

$$M = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -12X_0^2V_1 + 4\lambda^2V_1 - \frac{3V_1}{\eta_0^2} & 0 & \frac{6X_0V_1}{\eta_0^3} & 0 \\ 0 & 0 & 0 & -2\eta_0 \\ -12X_0V_1 & 0 & 8\eta_0^3 - \frac{3\sqrt{2}A^2\eta_0^2}{2} + \frac{3V_1}{\eta_0^3} & 0 \end{pmatrix}, \tag{22}$$

and Y is a column vector

$$Y = \begin{pmatrix} \tilde{x} \\ \tilde{\xi} \\ \tilde{\eta} \\ \tilde{\beta} \end{pmatrix}. \tag{23}$$

The solutions of this matrix equation can be written as

$$Y(t) = e^{E_n t}Y_n, \tag{24}$$

where E_n are the eigenvalues of the matrix M and Y_n are the corresponding eigenvectors. The stability of the fixed points depends upon the eigenvalues E_n. If any of the eigenvalues of M has a real part greater than zero, then the fixed points are unstable. The eigenvalues of M are given by

$$E_{1,2} = \pm\left(\frac{A'}{2} + \frac{BD}{2} - \frac{1}{2}\sqrt{(A'^2 - 2A'BD - B^2D^2 + 4BCE)}\right)^{1/2}, \tag{25}$$

$$E_{3,4} = \pm\left(\frac{A'}{2} + \frac{BD}{2} + \frac{1}{2}\sqrt{(A'^2 - 2A'BD - B^2D^2 + 4BCE)}\right)^{1/2}, \tag{26}$$

where

$$A' = -12X_0^2V_1 + 4\lambda^2V_1 - \frac{3V_1}{\eta_0^2}, \tag{27}$$

$$B = -2\eta_0, \tag{28}$$

$$C = -12X_0V_1, \tag{29}$$

$$D = 8\eta_0^3 - \frac{3\sqrt{2}A^2\eta_0^2}{2} + \frac{3V_1}{\eta_0^3}, \tag{30}$$

$$E = \frac{6X_0V_1}{\eta_0^3}. \tag{31}$$

With the above eigenvalues of the matrix M, we get that, if the expression inside the radical is positive, the fixed points are stable only if $X_0^2 < \lambda^2$, $X_0^2 \neq 0$. The condition for the positivity of the quantity under the square root yields

$$\eta_0 = \sqrt{\frac{3}{4\left(\lambda^2 - X_0^2\right)}}, \tag{32}$$

$$V_1 = \frac{9 - 10.392A^2\sqrt{\left(\lambda^2 - X_0^2\right)}}{32X_0^2\left(\lambda^2 - X_0^2\right)^2}. \tag{33}$$

Further, for a given value of ω, the positivity of η_0 and V_1 leads to a restriction on the allowed values of λ for the fixed points to be stable. In our case of $\omega = 0.1$, λ can vary only in the range approximately from 2.5 to 3.5. Therefore, in our numerical calculations, we have taken $\lambda = 3.0$ as an example. It corresponds to the minima of the double-well potential at $\pm 2.89\,\mu$m. The above set of conditions allows us to study different regimes of condensate dynamics by varying one or more parameters out of the set η_0, X_0, λ and A.

Numerical results and discussions

In this section we present some typical and interesting results for a condensate in a double-well potential alone. For this purpose, we numerically solve the system of equations (17)–(20) for different (three) values of the initial peak position of the condensate relative to the right well of the double-well potential, and show that the evolution of the condensate depends on the initial location of its peak.

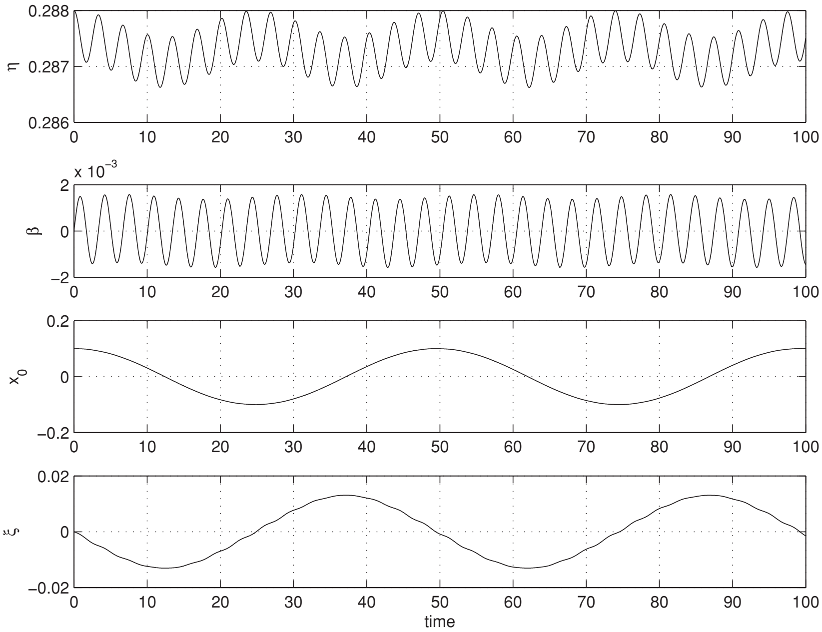
After fixing the value of the parameter λ, we take up a typical case of $A = 0.5$ (which corresponds to 2.5×10^5 atoms in the condensate) and determine the fixed points of the system for a set of values of X_0, which determines the initial peak position of the condensate relative to the right minimum of the double-well potential, situated at $\lambda = +3$. The table below contains the values of the relevant parameters for this case.

As we see, for $X_0 = 0.1$ (which means that the center position of the condensate is initially far away from the minimum of the right well), three fixed points, corresponding to the eigenvalues $E_1 = 1.8599i$, $E_2 = -1.8599i$ and $E_4 = -1.2435$ of the matrix M, are stable. The fixed point corresponding to $E_3 = +1.2435$ is unstable. Further, as we push the center position of the condensate toward the minimum of the right well ($X_0 = 2.1$ and 2.9), all four fixed points become stable.

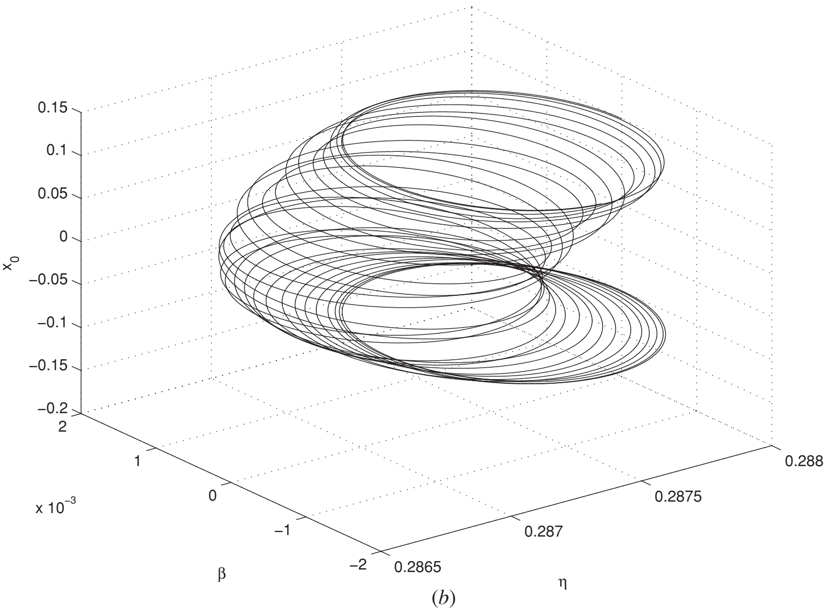
Case 1. This corresponds to $X_0 = 0.1$. The time evolution of the condensate parameters is shown in figure 1(a). As we see, the center position of the condensate oscillates between -0.1 and $+0.1$ which is clear from the three-dimensional representation of the condensate dynamics (in η, β and X_0) shown in figure 1(b). The amplitude of the condensate, η, and the chirp parameter, β, are periodic functions of time and are phase locked in the sense that they oscillate synchronously with a constant phase difference of $\pi/2$. This becomes evident from the fact that the projection of the phase space dynamics on the η–β plane [24], depicted in figure 1(c), is a circle. Further, it is clear from the 3d-plot of $|\psi|^2$, shown in figure 1(d), that the condensate remains confined during its evolution in the right well of the double-well potential (the well in which it is initially placed).

Case 2. Here, $X_0 = 2.1$, i.e., the peak of the condensate is now nearer to the minimum of the right well. The time evolution of the condensate parameters for this case is shown in figure 2(a). The amplitude, η, the chirp, β, and the center position x_0 have larger periods of oscillations compared to case 1. Unlike case 1, where the condensate remains confined in the well in which it is initially placed, here the condensate, as it evolves, undergoes tunneling from the right well into the left. In figure 2(b), where we have the three-dimensional plot of $|\psi|^2$, shows such a tunneling.

Case 3. Here $X_0 = 2.9$ and it corresponds to the case when the center position of the condensate almost coincides with the minimum of the right well. In figure 3 we have the

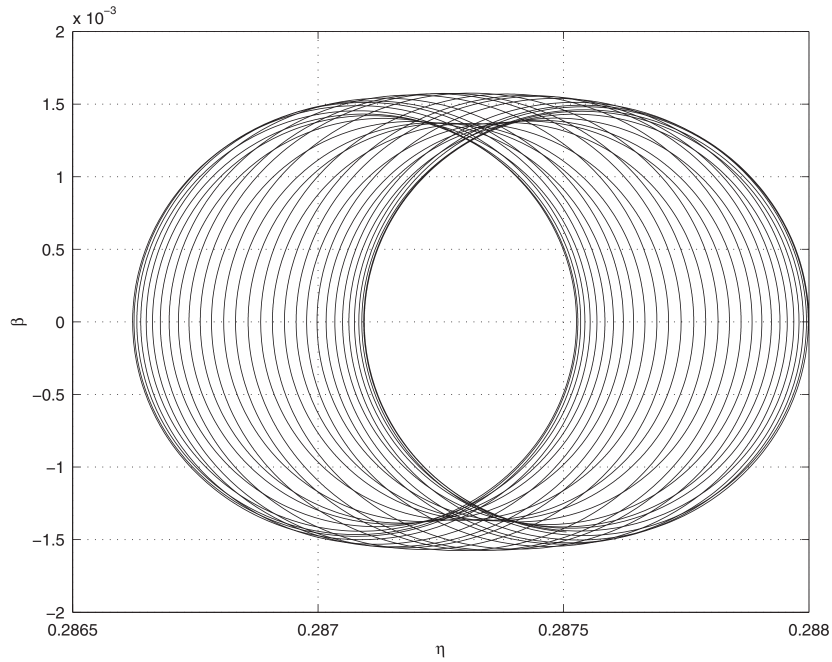


(*a*)

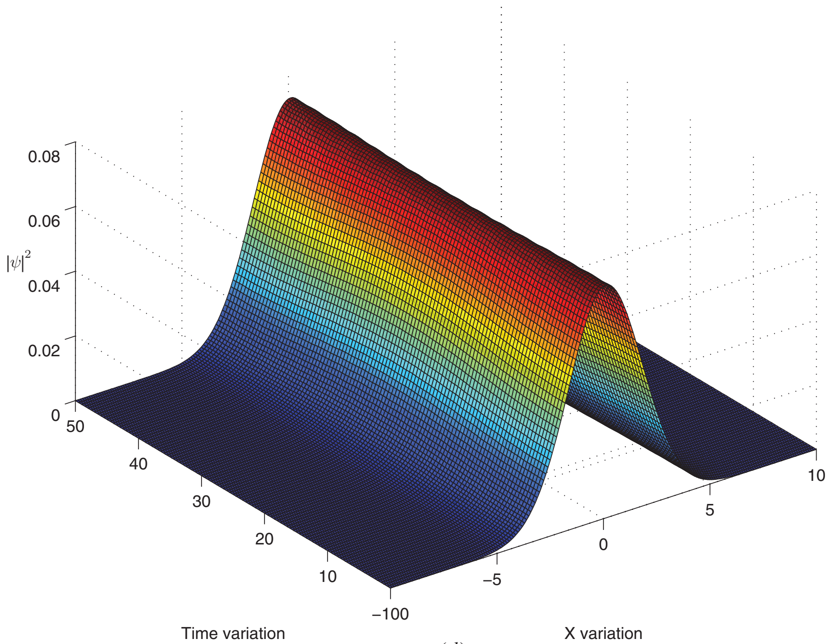


(*b*)

Figure 1. (*a*) Time evolution of the amplitude η, chirp β, center position X_0 and the frequency parameter ξ of the condensate, for $A = 0.5, \omega = 0.1, \lambda = 3$ and $X_0 = 0.10$, with the initial conditions $\eta_0 = 0.2880$ and $V_1 = 0.0468$, corresponding to case 1 given in table 1. (*b*) Three-dimensional representation of the condensate dynamics in (η, β, X_0) corresponding to the time evolution of the condensate parameters shown in figure 1(*a*). (*c*) The projection of the phase-space dynamics of the condensate (figure 1(*b*)) on the $\eta - \beta$ plane. (*d*) Three-dimensional plot of $|\psi|^2$ with the initial conditions of case 1 given in table 1. The condensate is initially placed in the right well of the double-well potential with peak at $X_0 = 0.1$.

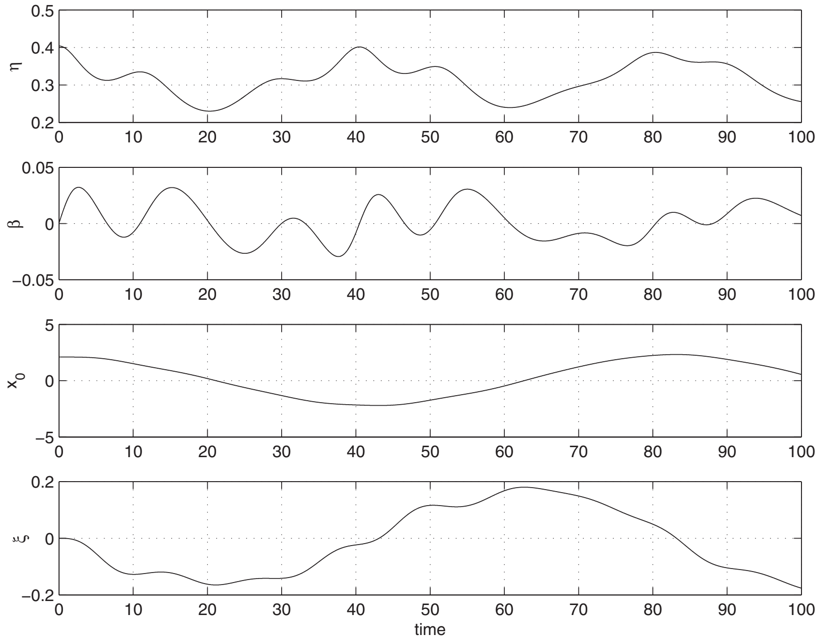


(*c*)



(*d*)

Figure 1. (Continued.)



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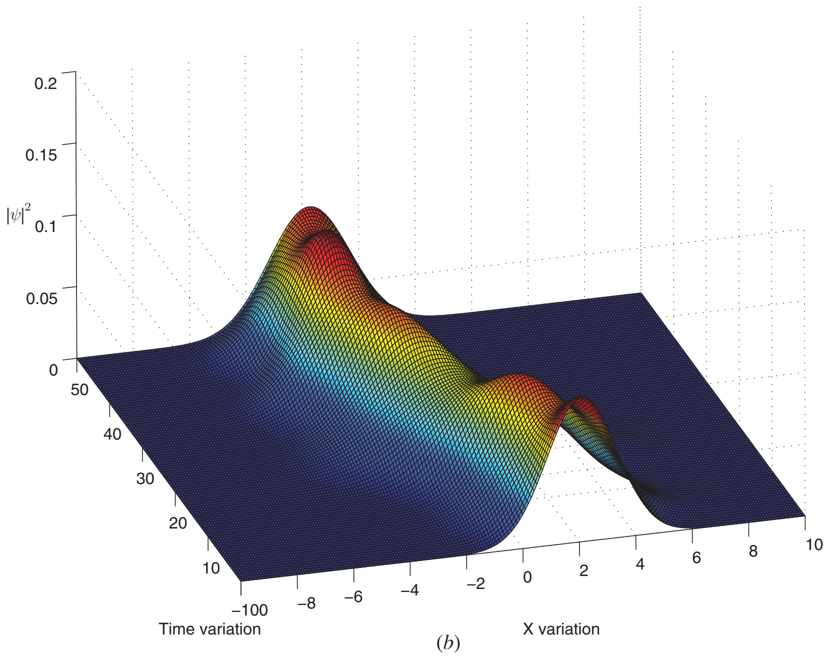


(*b*)

Figure 2. (*a*) The time evolution of the condensate parameters, for $A = 0.5$, $\omega = 0.1$ and $\lambda = 3$, with the initial conditions $X_0 = 2.1$, $\eta_0 = 0.4042$ and $V_1 = 0.0012$, corresponding to case 2 as given in table 1. (*b*) Three-dimensional plot of $|\psi|^2$, for $A = 0.5$, $\omega = 0.1$ and $\lambda = 3$, with the initial conditions $X_0 = 2.1$, $\eta_0 = 0.4042$ and $V_1 = 0.0012$, corresponding to case 2 as given in table 1.

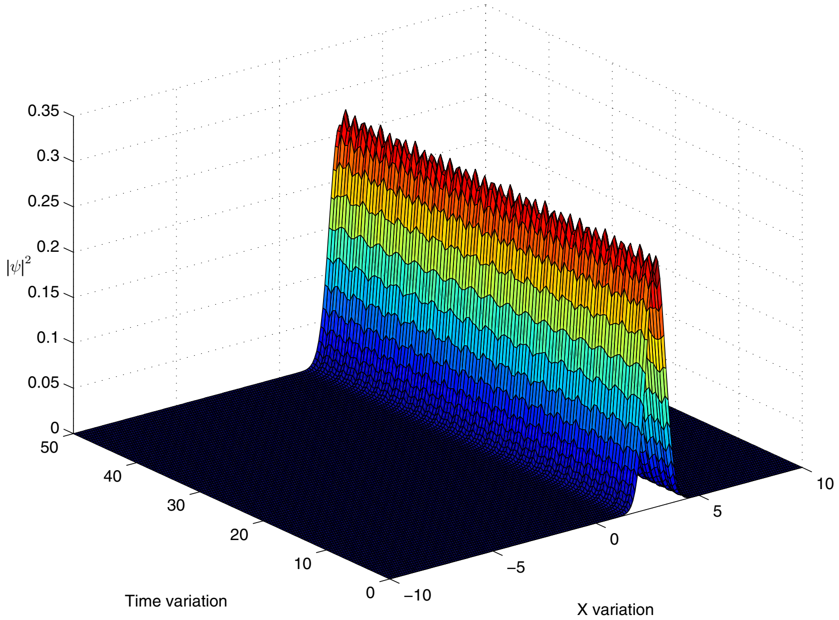


Figure 3. Three-dimensional plot of $|\psi|^2$, for $A = 0.5, \omega = 0.1$ and $\lambda = 3$, with the initial conditions $X_0 = 2.9$, $\eta_0 = 1.1275$ and $V_1 = 0.0748$, corresponding to case 3 as given in table 1.

three-dimensional plot of $|\psi|^2$, which shows that the condensate gets trapped in the right well and evolves in time without any significant change in its initial profile.

Thus, depending on the initial location in a given well of the double-well potential, the condensate can remain either trapped into the same well or tunnel into the neighboring well.

Condensate in a double-well potential perturbed by an optical lattice

Consider the evolution of a BEC under the combined influence of a double-well and a periodic potential given by equation (2). The reduced Lagrangian for this case is

$$\tilde{L} = A^2\sqrt{\frac{\pi}{2}}\left[\frac{1}{2\eta^2}\frac{\mathrm{d}\beta}{\mathrm{d}t} + 2\frac{\mathrm{d}\phi}{\mathrm{d}t} - 2\xi\frac{\mathrm{d}x_0}{\mathrm{d}t} + (\eta^4+\beta^2)\frac{1}{\eta^2} + \xi^2\right] + \sqrt{2\pi}A^2V_1\left\{\frac{3x_0^2}{2\eta^2} - \frac{\lambda^2}{2\eta^2} + \left(x_0^2-\lambda^2\right) + \frac{3}{16\eta^4}\right\} + A^2\sqrt{\frac{\pi}{2}}V_0\left(1-\mathrm{e}^{(-\omega^2/2\eta^2)}\right)\cos[2\omega(\bar{x}-x_0)] - \frac{A^4}{2}\eta\sqrt{\pi}. \tag{34}$$

Following Berry and Kutz [24], we assume that the width of the condensate is much smaller than the width of the crest of the periodic potential, i.e., $\omega/\eta \ll 1$, so that $\mathrm{e}^{(-\omega^2/2\eta^2)} \sim 1$. Under this assumption, we obtain the following set of reduced equations of motion:

$$\frac{\mathrm{d}\eta}{\mathrm{d}t} = -2\beta\eta, \tag{35}$$

$$\frac{\mathrm{d}x_0}{\mathrm{d}t} = \xi, \tag{36}$$

Table 1.

Parameters	X_0	η_0	V_1	Eigenvalues: 1, 2	Eigenvalues: 3, 4
Case 1	0.1	0.2880	0.0468	± 1.8599 i	± 1.2435
Case 2	2.1	0.4042	0.0012	± 0.6425 i	± 0.0594 i
Case 3	2.9	1.1275	0.0748	± 4.9955 i	± 2.1933 i

$$\frac{\mathrm{d}\beta}{\mathrm{d}t} = 2V_1\left(\lambda^2 - \frac{3}{4\eta^2} - 3x_0^2\right) + 2(\eta^4 - \beta^2) - \frac{A\sqrt{2}}{2}\eta^3 - V_0\omega^2\cos[2\omega(\bar{x} - x_0)], \tag{37}$$

$$\frac{\mathrm{d}\xi}{\mathrm{d}t} = 4V_1x_0\left(\lambda^2 - x_0^2\right) - \frac{3V_1x_0}{\eta^2} + V_0\omega\sin[2\omega(\bar{x} - x_0)]. \tag{38}$$

The fixed points of the above system of equations are given by $\eta = \eta_0$, $\beta = 0$, $x_0 = X_0$ and $\xi = 0$, where η_0 is determined from the equation

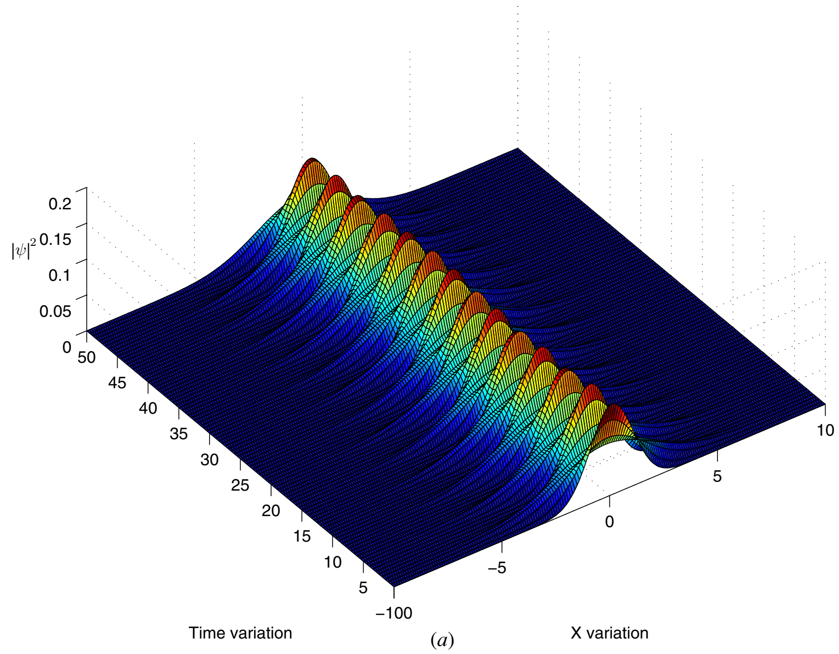
$$\eta^4 + \frac{A}{\sqrt{2}}\eta^3 - \frac{V_0\omega}{4x_0}(2\omega x_0\cos\theta + \sin\theta) - 2V_1x_0^2 = 0, \tag{39}$$

where $\theta \equiv 2\omega(\bar{x} - x_0)$. Since we want $x_0 \equiv X_0$ to be a free parameter in the sense that it gives us the freedom to place the initial BEC at a desired position in the well, the above equation can be solved for η, for a given A and the ratio (V_0/V_1). The real and positive roots of this equation will determine the steady state value of the condensate amplitude, η_0, for a fixed center position and a given relative strength of the periodic potential with respect to the double-well potential. In order to study the dynamics of this system as a function of the relative strength of the periodic and double-well potentials, we solve equation (39) for $X_0 = 0.1$, 2.1 and 2.9 and the values of V_1, given in table 1. The solution shows that only one root is real and positive and gives the required η_0.

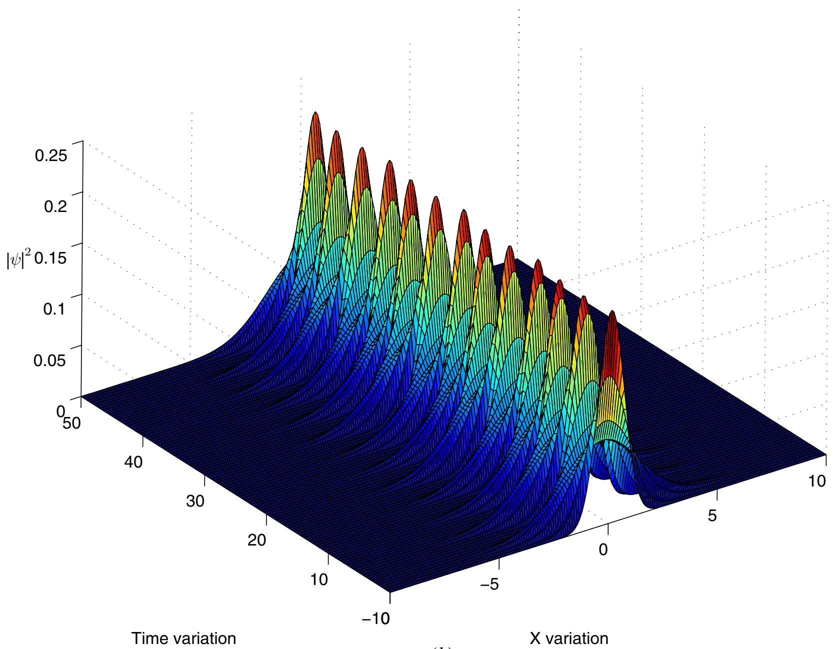
Having determined the fixed points, $(\eta_0, 0, X_0, 0)$, of the system of equations (35)–(38), we solved this system numerically for various values of the ratio (V_0/V_1) but with the same initial conditions corresponding to the new fixed points.

The three-dimensional plots of $|\psi|^2$, when the condensate is initially placed with its peak at $X_0 = 0.1$, are shown in figures 4(*a*) and (*b*) for $(V_0/V_1) = 100$ and 1000, respectively. We see that as time goes on the condensate hops between the original fixed point with the center position at $X_0 = 0.1$ and the newly generated fixed point with the center position at $X_0 = -0.1$. This results in a breathing dynamics of the condensate with a sequential narrowing and broadening of the intensity profile. The intensity with larger peak value corresponds to a greater confinement of the condensate near the newly generated fixed point which happens to be closer to the minimum of the double-well potential. The breathing dynamics is consistent with the results of the linear stability analysis.

The three-dimensional plots of $|\psi|^2$, for $X_0 = 2.1$, are shown in figures 5(*a*)–(*c*) for $(V_0/V_1) = 100$, 1000 and 5000, respectively. The dynamics here changes and a new feature emerges: the condensate starts tunneling back and forth between the two wells of the double-well potential. As the relative strength of the periodic potential is increased, the period of this tunneling oscillations decreases. The nature of dynamics for the third initial conditions with $X_0 = 2.9$ remains the same except that this period coupling between the wells decreases further. A typical result is shown in figure 6.

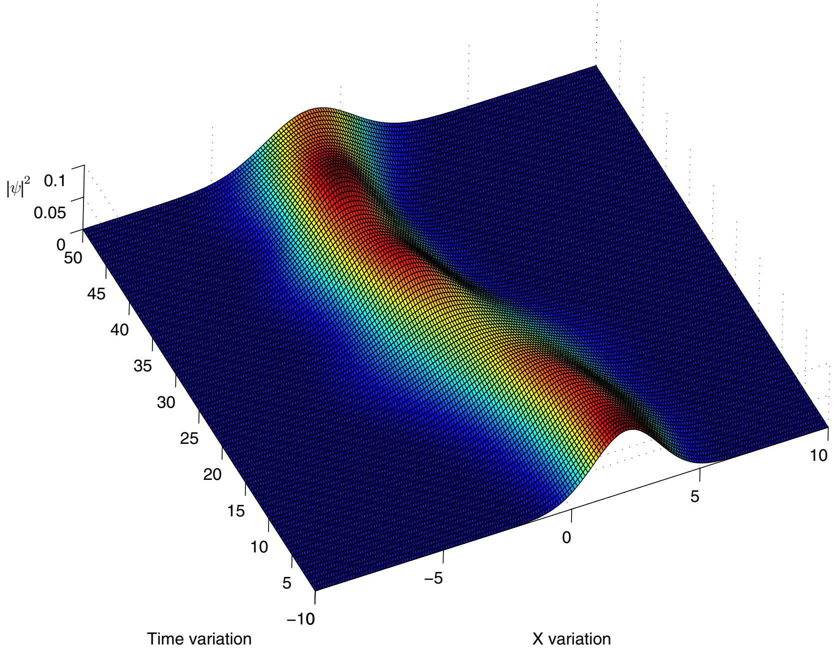


(*a*)

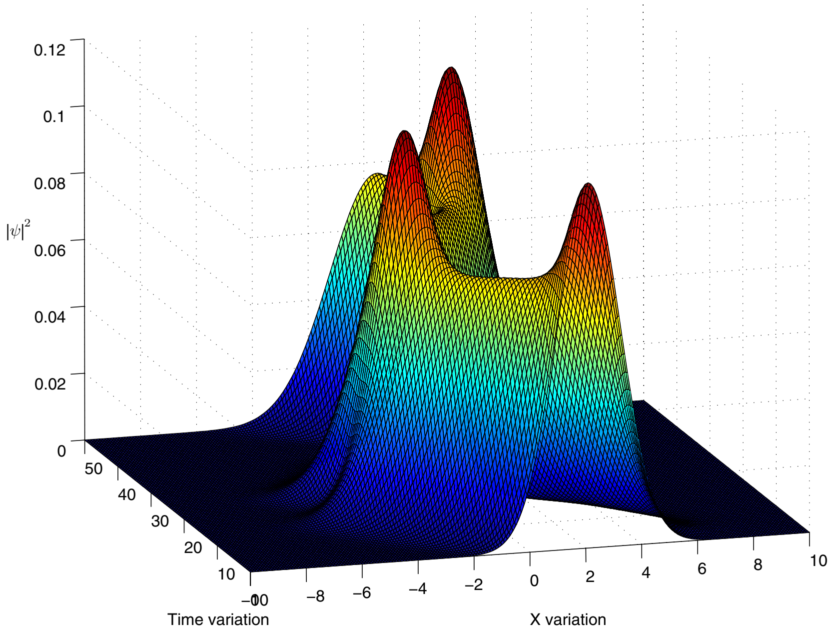


(*b*)

Figure 4. (*a*) The 3D plot of $|\psi|^2$ showing the breathing time evolution of the condensate in the double-well potential perturbed by a periodic potential for $V_0/V_1 = 100$ and the initial position of the peak at $X_0 = 0.1$. (*b*) The 3D plot of $|\psi|^2$ showing the breathing time evolution of the condensate in the double-well potential perturbed by a periodic potential for $V_0/V_1 = 1000$ and the initial position of the peak at $X_0 = 0.1$.

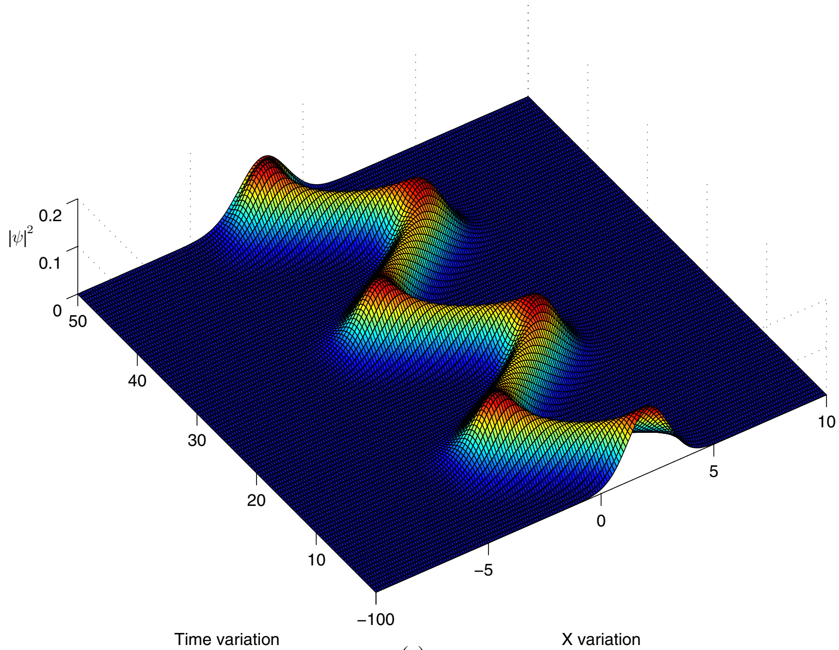


(*a*)



(*b*)

Figure 5. (*a*) The 3D plot of $|\psi|^2$ showing periodic coupling of the condensate between the wells of the double-well potential for $V_0/V_1 = 100$ and the initial position of the peak at $X_0 = 2.1$. (*b*) The 3D plot of $|\psi|^2$ showing periodic coupling of the condensate between the wells of the double-well potential for $V_0/V_1 = 1000$ and the initial position of the peak at $X_0 = 2.1$. (*c*) The 3D plot of $|\psi|^2$ showing periodic coupling of the condensate between the wells of the double-well potential for $V_0/V_1 = 5000$ and the initial position of the peak at $X_0 = 2.1$.



(*c*)

Figure 5. (Continued.)

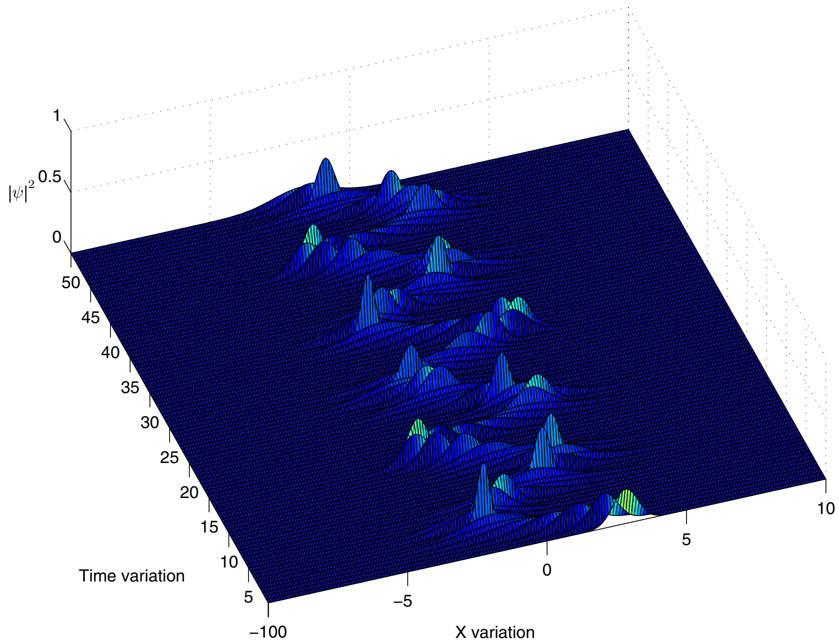


Figure 6. The 3D plot of $|\psi|^2$ showing periodic coupling, with shorter period, of the condensate between the wells of the double-well potential for $V_0/V_1 = 5000$ and the initial position of the peak at $X_0 = 2.9$.

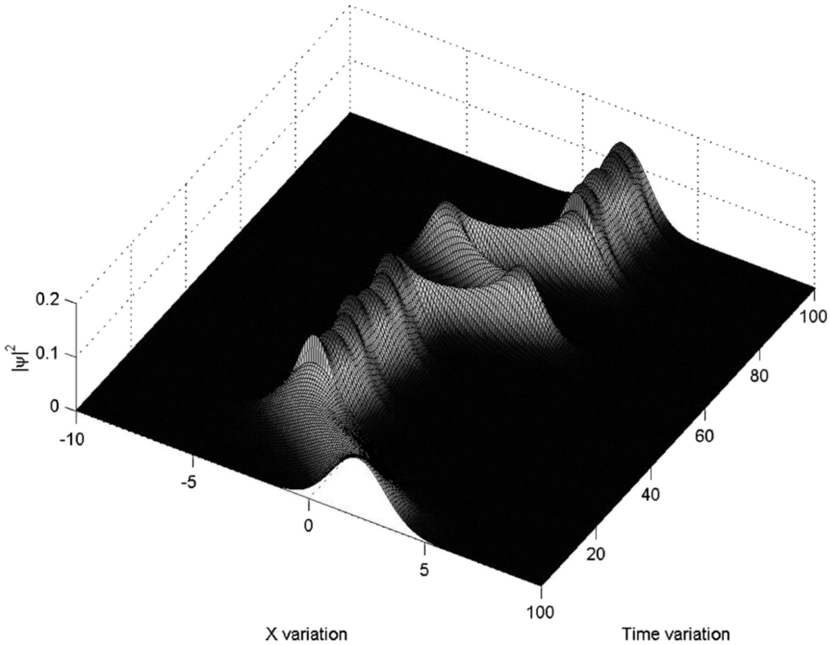


Figure 7. The 3D plot of $|\psi|^2$ corresponding to the time evolution of the condensate between the wells of the double-well potential with the initial conditions $\eta_0 = 0.4042$, $\beta = 0$, $\xi = 0$, $X_0 = 2.1$ and $V_1 = 0.0012$, for $V_0/V_1 = 1000$. The dynamics corresponds to the manipulation regime, as explained in the text.

Manipulating the BEC

We have seen earlier that, depending on the initial conditions, the BEC either remains confined in a given well or tunnels back and forth between the wells and that the period of tunneling oscillations depends on the relative strength of the periodic potential with respect to the double-well potential. This gives us an opportunity to control and manipulate the BEC the way we want. In what follows, we show how one can do this effectively by choosing the initial location of the BEC and varying the relative strength of the trapping potential. The idea here is similar to the one used in [24] for calculating a specific periodic potential for trapping the condensate in one of the fixed points.

In figure 7 we have the plot of $|\psi|^2$ for the case when initially the condensate is placed with its peak at $X_0 = 2.1$, $V_1 = 0.0012$ and the ratio $V_0/V_1 = 1000$. After a time interval of 20 units, when the condensate tunnels into the left well, the periodic potential is switched off and the strength of the double-well potential is increased to 0.0115, which corresponds to shifting of the minima of the double-well potential from $\lambda = 3$ to $\lambda = 2.5$. The subsequent time evolution shows that the condensate remains in the left well after tunneling. Further, we let the condensate evolve for another 30 units of time and then switch on the periodic potential with a larger relative strength, $V_0/V_1 = 5000$, and restore the strength of the periodic potential back to 0.0012 (this is equivalent to restoring the minima of the double-well potential back to $\lambda = 3$). We then let the condensate evolve in time. As we can see in figure 7, in its subsequent evolution the condensate starts tunneling back and forth between the wells of the double-well potential with a lesser period. Here, we would like to mention that, although our study is

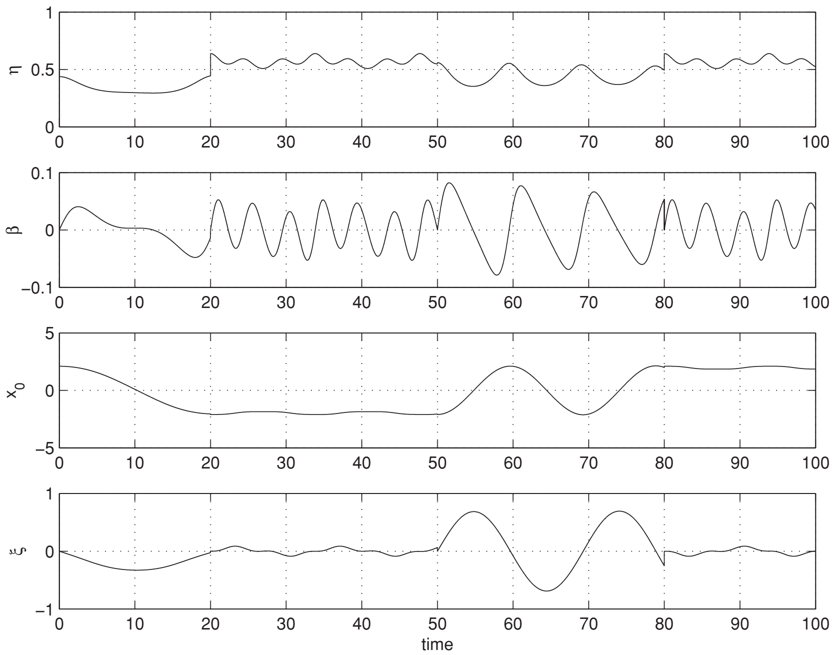


Figure 8. The time evolution of the condensate parameters η, β, X_0 and ξ during the manipulation regime with the initial conditions $\eta_0 = 0.4042$, $\beta = 0$, $\xi = 0$, $X_0 = 2.1$ and $V_1 = 0.0012$, for $V_0/V_1 = 1000$.

different from what is carried out in [25], the decrease in the period of oscillation between the wells may be similar to the fast Josephson and Rabi oscillations due to the excitation of quasiparticles, observed in [25]. After a time interval of 30 units, when the condensate is in the right well, we switch off the periodic potential and increase the strength of the double-well potential to 0.0115. The condensate now gets trapped in the right well and remains in it during its further evolution. In figure 8, we have presented the plots of the parameters of the condensate as a function of time during the manipulating process. Thus, we see that by suplementing the double-well potential with a periodic potential we can control and manipulate a BEC effectively.

Conclusions

In the given work we have carried out a variational analysis of the time evolution of a Bose–Einstein condensate in a double-well potential which is perturbed by a periodic potential (optical lattice). We have shown that the presence of optical lattice leads to a variety of possibilities for controlling and manipulating a BEC. For instance, depending on the relative strength of the periodic potential with respect to the double-well potential, new steady states are generated with varied degrees of confinement. We have also shown that by switching the optical lattice on and off and by adjusting the minima of the double-well potential appropriately, we can trap the BEC in a given well for any desired interval of time, make it tunnel through to the other well at will, or let it oscillate between the well with a desired period of oscillation.

Acknowledgments

We express our sincere thanks to our referees whose critical and constructive comments helped us in improving the presentation of the work and get rid of the shortcomings which were initially present in the manuscript.

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